The Active Bijection in Graphs, Hyperplane Arrangements, and Oriented Matroids 1. The Fully Optimal Basis of a Bounded Region. Erratum

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In [1], Proposition 5.1 and Theorem 5.3 do not hold as stated. In both, the hypothesis that $p < f$ are the first two elements of $B_{\min}$ should be replaced by the hypothesis that $p < f$ are the first two elements of E. Under this new hypothesis, proofs are valid without change[1]. However, to avoid a possible confusion with the notation $B_{\min} = \{p, f, \ldots\}_<$ used throughout the paper, we should rather write $E = \{e_1, e_2, \ldots\}_<$.

Proposition 5.1 *Let M be an ordered matroid on a set $E = \{e_1, e_2, \ldots\}_<$. A basis B of M is internal and uniactive if and only if $(E \setminus B) \cup \{e_1\} \setminus \{e_2\}$ is internal and uniactive in M^*.*

Theorem 5.3 *Let M be a bounded acyclic ordered oriented matroid on a set $E = \{e_1, e_2, \ldots\}_<$. We have*

$$\alpha(-_{e_1} M^*) = \big(E \setminus \alpha(M)\big) \cup \{e_1\} \setminus \{e_2\}.$$

The duality property in Theorem 5.3 is called the *active duality*. In the last part of Section 5, when comparing active duality to linear programming duality, it is implicitly assumed that $p = e_1$ and $f = e_2$, implying that $\{e_1, e_2\}$ is independent.

[1] E. Gioan, M. Las Vergnas, *The active bijection in graphs, hyperplane arrangements, and oriented matroids 1. The fully optimal basis of a bounded region*, European Journal of Combinatorics **30 (8)** (2009), 1868–1886.

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[1] Independently, in line 10 of the proof of Proposition 5.1, instead of $B' - f$ read $(E \setminus B') \setminus \{f\}$.